Brane-Universe in Six Dimensions with Two Times

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Abstract

Brane-Universe model embedded in 6-dimensional space-time with the signature (2+4) is considered. A matter is gravitationally trapped in three space dimensions, but both time-like directions are open. Choosing of the dimension and the signature of the model is initiated with the conformal symmetry for massless particles and any point in our world can be (1+1) string-like object.

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In conventional Kaluza-Klein's picture extra dimensions are curled up to unobservable size. Last years models with large extra dimensions become popular (see for example [1, 2]). Those approaches also do not contradict to present time experiments [3]. Main obstacles in progress of such models were how to confine a matter inside brane-Universe and to explain observed 4-dimensional Newton's law. In the papers [4, 5, 6, 7] we had introduced possible mechanism of overcoming these problems. Special solution of multi-dimensional Einstein's equations [4, 7, 8], responsible for gravitational trapping of a matter, provides effective 4-dimensional Newton's law on the brane. In this model it can be explained also non-observation of cosmological constant in four dimensions [4, 7, 9].

In our previous papers [4, 5, 6, 7] for simplicity 5-dimensional model with the signature (1+4) was considered. In general the question about the number of dimensions and signature is open. We consider that a matter is trapped on the brane by gravitation. So it is natural to assume that for massless case (it means weakest coupling with gravity) symmetries of sub-manifold can be restored. It is well known that in the zero-mass limit main equations of physics are invariant under fifteen parameter nonlinear conformal transformations. From the other hand a long time ago it was discovered that conformal group can be written as a linear Lorentz type transformations in 6-dimensional space-time with the signature (2+4) (for these subjects see for example [10]). Another indication that real number of space-like dimensions of Universe can be four, could be $O(4)$ symmetry of the solution of Shrodinger equations for the hydrogen atom.

Theories with extra time-like dimensions have been a subject of interest for some time (the latest article about large extra time-like dimensions is [11], see also [12, 13]). Kaluza-Klein 6-dimensional model with two times, but with compact extra dimensions, was investigated before in [14, 15]. Necessity of two times follow also from string theory (F-theory) [16]. There exist another two groups of 6-dimensional schemes, but with three times. The schemes of the first group [17, 18, 19] suffer from internal inconsistencies, second [20], more sophisticated scheme is internally consistent.

In all these papers multi time dimensions are introduced, however, not much is known

about the classical vacuum solutions of two- or more-timing theories. It is known that theories with compact internal time-like dimensions have several pathological features. The most conspicuous may be the fact that excitations of the internal dimensions have negative norm. Experimental lower bounds on possible violation of unitarity put a limit on the maximum radius of the internal time-like directions [21].

After unification of space and time coordinates (but not dimensions) in Lorents transformations many physicists intuitively are accustomed to consider space and time dimensions identically. However, there is principal difference among them. Main difference is that one can easily change position, or stop in the space but everybody follows to time flow which was began from the Big Bang. Decreasing of time flow, or shift of time vector for any system, means backward moving in the time and then disappearance for other observers. Another problem is that we know several space directions and it is easy to add next ones, but we know only one time. In case of two or more time directions the question arises what we are measuring with our clocks. Thus ordinary methods of trapping using in our previous papers [4, 5, 6, 7] can be not applicable for time-like dimensions.

Possible indication of existence of extra time directions can be non-conservation of the energy in four dimensions. This can be happening only after interaction with the matter with another direction of time (or energy) vector. As we mentioned above we only passively follow to time flow, direction of which coincides with cosmological arrow of time (possible source of different asymmetries in our world, see for example [24]). So it could work mechanism similar to that considered in papers [20] - after some time from the Big Bang all particles with another directions of time had been disappeared from our view. In this case may be no special trapping mechanism in extra times is necessary.

All points of the space in Universe are equivalent. However, we have global zero for time coordinate, it is the moment of the Big Bang. What we measure for any particle is the difference of two energies - energy of the vacuum and the particle itself. When particle follows to our time flow we notice only this difference, but to shift the time vector we need total value of the energy corresponding to the age of Universe. Thus if the particles with another energy vector had been disappeared the matter of our world is trapped in our time by Plank scale.

In this paper we consider 6-dimensional Kaluza-Klein model with one space-like and one time-like extended extra dimension. It is generalization of 5-dimensional scheme of papers [4, 5, 6, 7] with one brane where all matter is localized. Trapping mechanism considered in these articles works for the case of one extra space-like dimension and is applicable for 6-dimensional model with two times. Time-like dimensions in our scheme are open, similar to the approach of [20].

We are looking for solution of 6-dimensional Einstein's equations with the cosmological term Λ in two time- and four space-like dimensions

$$ {}^6R_{AB} - \frac{1}{2} g_{AB}\, {}^6R = -\Lambda g_{AB} + G\tau_{AB} \quad . \tag{1} $$

Here ${}^6R_{AB}$ and G are 6-dimensional Ricci tensor and gravitational constant and big Latin indices $A, B, ... = 0, 1, 2, 3, 5, 6$.

Energy-momentum tensor for brane-Universe with the signature (2+3) embedded in 6-

dimensional space-time with signature (2+4) is taken in the form

$$\tau_{\mu\nu} = g_{\mu\nu}\sigma\delta(\frac{x^6}{\epsilon}), \quad \tau_{55} = \sigma\delta(\frac{x^6}{\epsilon}), \quad \tau_{66} = 0 \quad . \tag{2}$$

Here σ is brane tension, $\delta(x^6/\epsilon)$ - delta function and $\epsilon \sim \sqrt{\Lambda}$ - brane width in extra space-like dimension x^6. Greek indices $\alpha, \beta ... = 0, 1, 2, 3$ numerate coordinates in four dimensions. This form of the brane energy-momentum follows from kink-like solution of scalar field equations without introducing of coupling with gravity. In general it is difficult to separate the energy of the brane and gravitation field and we must consider more complicate model.

In (1) we choose negative sign for cosmological constant Λ. Canceling mechanism of papers [4, 5, 6, 7] for space-like extra dimension works only for the negative Λ. Solution with positive Λ corresponds to trapping in extra time and is considered at the end of the paper.

In many Kaluza-Klein models with large extra dimensions localization of particular quantum fields on the brane is investigated (latest paper in this direction is [22]). In our model [4, 5, 6, 7] matter is trapped on the brane by gravitation. The source of anti-gravity responsible for this trapping can be negative multi-dimensional cosmological constant. Since gravity is universal field we don't need any other classical source for localization of a matter on the brane. Exact mechanism for different fields is difficult to find because of problems with consideration of quantum fields in curve space-time (see for example [23]). Even in four dimensions only few exactly solvable models exist. Our approach is more general. Gravitation field is not localized on the brane, however, by canceling mechanism we have ordinary Newton's law. Gravitational potential has minimum on the brane and all particles having coupling with gravity are sitting there.

To keep the width of brane-Universe during expansion, it means for the stabile localization of the mater on the brane, as in paper [6] we look for the solution of (1) with zero extra momentum

$$P_i = \int T_i^A dS_A = 0 \quad . \tag{3}$$

Small Latin indices $i, j, k, ... = 5, 6$ numerate coordinates of extra dimensions. Here $T_A^B = t_A^B + \tau_A^B$ is total energy momentum tensor of matter fields τ_A^B and gravitational field itself

$$t_A^B = \frac{1}{2G}[g^{BD}\partial_A\Gamma^E_{DE} - g^{ED}\partial_A\Gamma^B_{DE} + \delta^B_A(\,^6R - 2\Lambda)] \quad . \tag{4}$$

Type of matter fields is not important now. Some particular cases are considered in the paper [6].

In this article we don't want to touch the old problem with the energy of gravitation and as in the paper [6] choose T_A^B in the form of so called Lorentz energy-momentum complex

$$T_A^B = \frac{1}{2G\sqrt{g}}\partial_C X_A^{BC} \quad , \tag{5}$$

where

$$X_A^{BC} = -X_A^{CB} = \sqrt{g}[g^{BD}g^{CE}(\partial_D g_{AE} - \partial_E g_{AD})] \quad . \tag{6}$$

This form is convenient, since in this case energy-momentum tensor of gravitational field t_A^B coincides with canonical energy-momentum tensor (4) received from Hilbert's form of the gravitational Lagrangian $L_g = \sqrt{g}(\,^6R - 2\Lambda)$.

To satisfy the stability condition (3), components of the energy-momentum tensor on the solutions must satisfy the condition

$$T_i^A = 0 \quad . \tag{7}$$

From $(i\alpha)$ component of this relation and (4) we find $\partial_i \Gamma^\alpha_{\mu\nu} = 0$. Thus simple solution of (7) is

$$g_{i\alpha} = 0 \quad , \quad g_{\alpha\beta} = \lambda(x^i)\eta_{\alpha\beta}(x^\nu) \quad , \tag{8}$$

where $\eta_{\alpha\beta}(x^\nu)$ is ordinary 4-dimensional metric tensor and $\lambda(x^i)$ is arbitrary function of extra coordinates. Solution (8) which we received from stability conditions, is similar with the anzats of [9].

Stability condition (7) for the case of diagonal metric tensor of extra dimensions ($g_{56} = 0$) has the form

$$\partial_5(g^{55}g^{66}\partial_5 g_{66}) = \partial_6(g^{55}g^{66}\partial_6 g_{55}) = 0 \quad . \tag{9}$$

One of the solutions of this system is

$$g_{55} = exp(cx^6) \quad , \quad g_{66} = -1 \quad , \tag{10}$$

where c is integration constant.

Using (8) one can find decomposition of Einstein's equations (see also [2])

$$\begin{aligned} R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}(D_iD^i\lambda + \frac{1}{2\lambda}D_i\lambda D^i\lambda) &= \frac{1}{2}g_{\alpha\beta}[\Lambda - \frac{1}{2}G\sigma\delta(\frac{x^6}{\epsilon})] \quad , \\ R_{ij} - \frac{2}{\lambda}(D_iD_j\lambda - \frac{1}{2\lambda}D_i\lambda D_j\lambda) &= \frac{1}{2}g_{ij}[\Lambda - \frac{5}{2}G\sigma\delta(\frac{x^6}{\epsilon})] \quad . \end{aligned} \tag{11}$$

Here R_{ij} and D_i are correspondingly Ricci tensor and the covariant derivative in extra space-time with the metric tensor g_{ij}. Ricci tensor in four dimensions $R_{\alpha\beta}$ is constructed from $g_{\alpha\beta} = \lambda(x^i)\eta_{\alpha\beta}(x^\nu)$ in a standard way.

Using (10) and the properties of the step function $H(x^5)$

$$|x|' = H(x) - H(-x) \quad , \quad H(x)' = \delta(x) = \frac{1}{|\epsilon|}\delta(\frac{x}{\epsilon)} \tag{12}$$

where prime denotes derivative, one can show that system (11) has the trapping solution

$$\lambda = g_{55} = exp(c|x^6|) \quad , \quad g_{66} = -1 \quad . \tag{13}$$

The integration constant here has the value

$$c = \sqrt{\frac{2\Lambda}{5}} = -\frac{G\sigma\epsilon}{4} \quad . \tag{14}$$

This formula also contains necessary relation between the brane tension σ and 6-dimensional cosmological constant Λ.

For this solution in four dimensions we have ordinary Einstein's equations without the cosmological term

$$R_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}R = 0 \quad , \tag{15}$$

which is function of only 4-dimensional metric tensor $\eta_{\alpha\beta}(x^\nu)$. After adding of 4-dimensional source at the right hand of (15) one can show that as in 5-dimensional case [4, 7] a matter is trapped in three space with ordinary Newton's low, while now we have unobservable extra open time direction.

From (14) we see that constant c is real only for our choice of sign of Λ. Also we noticed that for branes with positive tension constant c is negative and function λ decreasing fare from the brane, as in papers [2]. For the case of negative σ exponential factor in (13) is positive as in papers [4, 7, 8] and gravitational potential has minimum on the brane.

In this paper and our previous articles [4, 5, 6, 7], in contrast with the approach of [2], we consider only one brane. Interactions of branes with the negative and positive tensions are often considered in Kaluza-Klein theories with the large extra dimensions. One must be careful in this case. It is known for a long time [25] that system of negative and positive masses began to accelerate till the speed of the light. Acceleration can destroy the branes. Even one brane with positive tension is strange objects, since it is gravitationally repulsive [26]. This can cause change of the time direction on the brane [27], while t^2 is still positive. Negative tension can change signature and thus interchanges time and space coordinates.

System (11) with positive Λ has similar to (13) solution

$$\lambda = -g_{66} = exp(c|x^5|) \quad , \quad g_{55} = 1 \quad , \tag{16}$$

which corresponds to trapping in extra time, while all four space-like coordinates are open.

At the end of the paper we want to note that in this paper 6-dimensional model with string-like extra dimension was considered. Any point-like particle in our world can have tail in (1+1) dimensions and we have interesting possibility for nontrivial application of string theory.

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